

INTERVAL MARKOVIAN MODELS IN
DEPENDABILITY EVALUATION

Sérgio Galdino¹, Paulo Maciel^{2 §}, Nelson S. Rosa³

¹Department of Statistics and Informatics – DEI

Catholic University of Pernambuco

Recife, 50 050-900, BRAZIL

e-mail: segal@unicap.br

^{2,3}Informatics Center – CIN

Federal University of Pernambuco

Recife, 50 732-970, BRAZIL

²e-mail: prmm@cin.ufpe.br

³e-mail: nsr@cin.ufpe.br

Abstract: Model-based dependability evaluation is based on abstractions of the real system. When uncertainties or variabilities are associated with system parameters, single point characterization of parameters is inadequate. Interval arithmetic has been applied to model uncertainties and variabilities. An interval model is a space, family or class of models in which there are parameters represented by intervals, instead of real numbers. This paper describes the Interval Generalized Stochastic Petri Net (IGSPN) as an interval extension to the GSPN model. The IGSPN analysis takes into account the effects of variability on exponential transition rates and weights when calculating dependability measures. IGSPN analysis may be useful as a tool for decision-making. A case study related to availability evaluation of two network devices widely used in communication networks, namely a Multiplexer ADM and a Multiplexer SDH.

AMS Subject Classification: 93E03

Key Words: interval models, stochastic modeling, dependability measures, stochastic Petri Nets

Received: March 13, 2007

© 2007 Academic Publications

§Correspondence author

1. Introduction

To stay in business, organizations must develop and maintain a value proposition that is compelling, whether you are a high-volume or a low-volume provider of products or services. Customers expect receiving products that work properly for long as well as also expect continuous delivering of contracted services. Over the last decade, dependability issues have become a critical aspect of that proposition [8].

The successful implementation of dependable systems encompasses several aspects within an organization: from business infrastructure to continuous personal educational capacity programs; from quality assurance management methods to tools and models for evaluation of quality issues related to the products or services delivered.

The development of advanced technologies, methods and tools for dependability evaluation are fundamental for allowing the analysis of energy reliability and availability service levels.

Consider a system operating in environments that are driven by uncertainties and variabilities. This happens especially when dealing with DES (Discrete Event Systems) that, by their characteristics, often involve uncertain human interactions and unpredictable machine failures. If we are to derive quantitative means of timed DES analysis for performance/dependability evaluation with respect to quantitative performance measures in the presence of uncertainty, models that incorporate interval stochastic elements are required. ICTMC (Interval Continuous Time Markov Chains) is a formalism basis that allows incorporating interval arithmetic into Markovian framework.

Petri nets are powerful tools for description and analysis of discrete event systems (DES) that exhibit concurrency, synchronization and conflicts. The Generalized Stochastic Petri Net (GSPN) is one of the most applied timed Petri nets models for performance and dependability evaluation [28, 10, 44].

The availability of software tools for performance and dependability evaluation allows one to hide the Markov chain technicalities from the end-user. Users specify their performance model using some high-level modeling language, such as queueing networks (PEPSY-QNS), stochastic Petri nets (*TimeNET 4.0*, *SPNP 6.0*, *GreatSPN 2.0*) or stochastic process algebras (PEPA) from which the underlying Markov chain can be automatically generated and analyzed. High-level specification techniques for Markov chains are an active area of research [4].

Model-based dependability evaluation is based on abstractions of the real

system. When uncertainties or variabilities are associated with system parameters, single point characterization of parameters is inadequate. Uncertainties may be associated with parameters that are not known in advance, especially in the early stages of system design. Nevertheless, designers may have a good idea about the range of values associated with these parameters, considering previous experiences with similar systems [24]. System parameter variabilities may represent different mean demands for a given system component during various time periods. Therefore, when parameters are not precisely known, but known to lie within a certain interval, algorithms may be implemented using interval arithmetic.

This work proposes IGSPN, a high-level modeling formalism for automatic generation of ICTMC, aiming at modeling and evaluating of system's dependability. IGSPN is a GSPN [29] extension, defined as a class of stochastic Petri nets, whose time and weight are represented by intervals. Therefore, the set of methods considered for steady analysis have to be adapted for taking into account interval arithmetic. This work uses the *MATLAB* toolbox *Intlab* to implement the steady state analysis algorithms.

Next section gives a brief introduction of dependability and interval arithmetic. Section 3 describes the IGSPN model as GSPN interval extension. Section 4 models a case study and presents the respective analysis. Section 5 presents some conclusions and future work.

2. Background

Before presenting the IGSPN model, this section introduces some basic concepts needed to understand how interval arithmetic is adopted for evaluating systems dependability. Hence, we initially introduce concepts of dependability followed by interval arithmetic elements.

2.1. Dependability

Dependability of a computer system is understood as the ability to deliver services that can justifiably be trusted [2]. Dependability combines reliability and availability, etc. [12].

Reliability is the conditional probability that the system has survived (it is operational) in a given time interval $[0, t]$, since it was operational at time $t=0$ [25]. This attribute measures the continuity of a correct service

delivering, from an initial time reference:

$$R(t) = 1 - F(t), \quad (1)$$

where $F(t)$ is the cumulative distribution function. Availability, on the other hand, is the probability that the system is operational considering the steady-state regime, i.e., the percentage of time of which the system is delivering the specified service:

$$A = UpTime / (UpTime + DownTime), \quad (2)$$

where *UpTime* and *DownTime* represent the operational time and non-operational time period, respectively.

Many different techniques have been adopted for increasing dependability of systems. Fault tolerance (redundancy) aims at allowing correct service delivering even in presence of faults. Redundancies are extra resources, not necessary for the execution of faultless tasks, but that must be applied in case of faults, in order to guarantee the continuity of the correct service delivering. The redundancy techniques can be static or dynamic, depending on if the faults are to be masked or detected, respectively [22]. In static redundant mechanism, the main element (device, task, service, etc.) and redundant elements are permanently active, hence in case of faults the users do not perceive it, even performance degradation. TMR (triple modular redundancy), the most usual static redundant structure, is composed of three redundant modules and a voter, which is responsible for evaluating majority result delivering. If one of the modules becomes faulty, the two remaining fault-free mask the result of the faulty module when majority voting is performed.

The dynamic redundancy (active redundancy) mechanisms are characterized by fault detection followed by recovering actions, so that extra processing time is needed. The hot standby sparing arrangement is a dynamic redundancy mechanism. In this method, a fault is first detected then a control mechanism switches the spare module (that is kept permanently powered) to take over the active module operations.

Many other static and active redundancy mechanisms have been considered for allowing systems' specified service delivering, even in presence of faults. Among them, *cold standby*, *recovery blocks* and *N-Version* programming are some of the most prominent.

2.2. Interval Arithmetic

The interval arithmetic was developed as tool to study errors limits in numeric computations [30]. It is used to determine the effects of initial data, round-off and truncation errors. The interval width indicates the result accuracy. Therefore, the great challenge is to produce narrow intervals so that they can be acceptable in practical applications. The simple adoption of interval arithmetic in classical numerical algorithms may lead to error bounds too pessimistic. Therefore, for applying interval arithmetic, the development of particular algorithms is fundamental.

2.2.1. Notation

Throughout this paper, all matrices are denoted by bold capital letters (\mathbf{A}), vectors by bold lowercase letters (\mathbf{a}), and scalar variables by ordinary lowercase letters (a). Interval variables are enclosed in square brackets ($[\mathbf{A}]$, $[\mathbf{a}]$, $[a]$). Underscores and overscores denote lower and upper bounds, respectively. Angle brackets $\langle \rangle$ are used for intervals defined by a midpoint and a radius.

A real interval $[x] = [\underline{x}, \bar{x}]$ is a nonempty set of real numbers $[x] = \{\tilde{x} \in \mathbb{R} : \underline{x} \leq \tilde{x} \leq \bar{x}\}$, where \underline{x} and \bar{x} are called the *infimum* (*inf*) and *supremum* (*sup*), respectively, and \tilde{x} is a point value belonging to an interval variable $[x]$.

The set of all intervals \mathbb{R} is denoted by $I(\mathbb{R})$, where

$$I(\mathbb{R}) = \{[\underline{x}, \bar{x}] : \underline{x}, \bar{x} \in \mathbb{R} : \underline{x} \leq \bar{x}\}.$$

An interval vector is defined to be a vector with interval components and the space of all n dimensional interval vectors is denoted by $I(\mathbb{R})^n$. Similarly, an interval matrix is a matrix with interval components and the space of all $m \times n$ matrices is denoted by $I(\mathbb{R})^{m \times n}$. A point matrix or point vector has components with zero radius only, otherwise it is said to be a thick matrix or thick vector. Arithmetical operations on interval vectors and matrices are carried out according to interval arithmetic operations.

The midpoint of $[x]$ is defined as, $mid[x] = \tilde{x} = \frac{1}{2}(\underline{x} + \bar{x})$ and the radius of x is defined as, $rad[x] = \Delta x = \frac{1}{2}(\bar{x} - \underline{x})$. The radius may be used to define an interval $[x] \in I(\mathbb{R})$. Then $\langle \tilde{x}, \Delta x \rangle$ denotes an interval with midpoint \tilde{x} and radius Δx . The $[x, x] \equiv x$ is called point interval or thin interval. A point or thin interval has zero radius and a thick interval has a radius greater than zero. The absolute value of $[x]$ is denoted by $|[x]| = \max\{|\underline{x}|, |\bar{x}|\}$

and the (Hausdorff) distance by $q([x], [y]) = \max\{|\underline{x} - \underline{y}|, |\bar{x} - \bar{y}|\}$. The elementary operations on intervals $[a] = [\underline{a}, \bar{a}]$ and $[b] = [\underline{b}, \bar{b}]$ are explicitly calculated as:

$$- [\underline{a}, \bar{a}] + [\underline{b}, \bar{b}] = \left[\underline{a} \overset{\nabla}{+} \underline{b}, \bar{a} \overset{\Delta}{+} \bar{b} \right]$$

$$- [\underline{a}, \bar{a}] - [\underline{b}, \bar{b}] = \left[\underline{a} \overset{\nabla}{-} \bar{b}, \bar{a} \overset{\Delta}{-} \underline{b} \right]$$

$$- [\underline{a}, \bar{a}] \cdot [\underline{b}, \bar{b}] = \left[\begin{array}{l} \min \left(\underline{a} \overset{\nabla}{\cdot} \underline{b}, \underline{a} \overset{\nabla}{\cdot} \bar{b}, \bar{a} \overset{\nabla}{\cdot} \underline{b}, \bar{a} \overset{\nabla}{\cdot} \bar{b} \right), \\ \max \left(\underline{a} \overset{\Delta}{\cdot} \underline{b}, \underline{a} \overset{\Delta}{\cdot} \bar{b}, \bar{a} \overset{\Delta}{\cdot} \underline{b}, \bar{a} \overset{\Delta}{\cdot} \bar{b} \right) \end{array} \right]$$

$$- [\underline{b}, \bar{b}]^{-1} = \left[1 \overset{\nabla}{/} \bar{b}, 1 \overset{\Delta}{/} \underline{b} \right] \text{ if } 0 \notin [\underline{b}, \bar{b}]$$

$- [\underline{a}, \bar{a}] / [\underline{b}, \bar{b}] = [\underline{a}, \bar{a}] \cdot [\underline{b}, \bar{b}]^{-1}$. ∇ and Δ denote downward and upward directed rounding, respectively. These operations are called interval arithmetic. Besides these operations, interval functions (exp, log, sin, cos, ...) return both upper and lower bounds.

The interval arithmetic operations are defined for exact calculation [30]. Machine computations are affected by rounding errors. Therefore, the formulas were modified in order to consider the called direct rounding [19].

One important function in *Intlab* (a software tool-box for interval arithmetic) is the function *setround* which allows setting the processor's rounding mode to the nearest, round down and round up [34]. This routine suits interval arithmetic and allows consistent implementation of numerous other functions.

An important result is the inclusion property, often called the fundamental theorem of interval analysis.

Theorem 1. (Fundamental Theorem) *If the function $f([x]_1, [x]_2, \dots, [x]_n)$ is an expression with a finite number of intervals $[x]_1, [x]_2, \dots, [x]_n \in I(\mathbb{R})$ and interval operations $(+, -, \times, \div)$, and if $[w]_1 \subseteq [x]_1, \dots, [w]_n \subseteq [x]_n$ then $f([w]_1, [w]_2, \dots, [w]_n) \subseteq f([x]_1, [x]_2, \dots, [x]_n)$.*

2.2.2. Interval Linear Systems

The solution to linear systems of equations is prone to errors due to the finite precision of machine arithmetic and error propagation related to the initial data. If the initial data is known to lie within specified ranges, then interval arithmetic enables computation of intervals containing the elements of the exact solution [15].

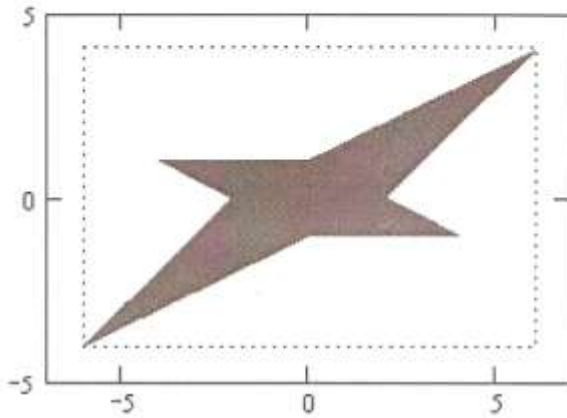


Figure 1: Solution set and hull of an interval linear system of equations

An interval linear system is of the form

$$[\mathbf{A}][\mathbf{x}] = [\mathbf{b}], \tag{3}$$

where $[\mathbf{A}] \in I(\mathbb{R})^{n \times n}$ and $[\mathbf{b}] \in I(\mathbb{R})^n$.

The solution set

$$\Sigma([\mathbf{A}], [\mathbf{b}]) = \left\{ \tilde{\mathbf{x}} : \tilde{\mathbf{A}} \cdot \tilde{\mathbf{x}} = \tilde{\mathbf{b}} \text{ for some } \tilde{\mathbf{A}} \in [\mathbf{A}], \tilde{\mathbf{b}} \in [\mathbf{b}] \right\}. \tag{4}$$

That is, $\Sigma([\mathbf{A}], [\mathbf{b}])$ is the set of all solutions of (3) for all $\tilde{\mathbf{A}} \in [\mathbf{A}]$ and all $\tilde{\mathbf{b}} \in [\mathbf{b}]$. The shape of solutions of an interval linear system can be fairly complicated, but since interval arithmetic has been used, only a multidimensional box is computed [23, 20, 31, 14] that contains the true solution set. For example, consider the system, where

$$\mathbf{A} = \begin{pmatrix} [1, 4] & [-1, 3] \\ [-1, 0] & [1, 3] \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} [-1, 1] \\ [-1, 1] \end{pmatrix},$$

then, the shaded area of Figure 1 shows the solution set of interval linear system of equations.

The optimal solution is the smallest box containing the true solution. This interval vector is called the *hull of the solution set* or simply the *hull* [31]. When saying “solving” the system, it is meant finding the hull $[\mathbf{x}]^H$. The problem of find the hull is known to be NP-Hard [18] and only hull’s

outer bounds are generally computed. Various iterative and direct methods have been published for accomplishing this task [31, 15, 14].

It is difficult to write (3) in an unambiguous way. Consider the simple case $[\mathbf{A}] = [1, 2]$ and $[\mathbf{b}] = [3, 4]$. The solution set is the interval

$$[\mathbf{x}] = \frac{[3, 4]}{[1, 2]} = \left[\frac{3}{2}, 4\right].$$

But $[\mathbf{A}] \cdot [\mathbf{x}] = [3/2, 8]$, which does not equal $[\mathbf{b}]$. That is, the solution cannot generally be substituted into the given equation and reaching an equality. All it could be asserted is that $[\mathbf{A}] \cdot [\mathbf{x}] \supset [\mathbf{b}]$ (see Theorem 1).

However, we insist of writing a "system of interval linear equations" in form (3) wherein $[\mathbf{x}]$ occurs as if was a real vector. The appropriate understanding is that the "equation" requires interpretation [14].

The optimal solution of an Interval System of Linear Algebraic Equations is of current interest [23]. The so-called *outer problem for an interval system of algebraic equations* [38, 37]: *find an interval vector with minimal width containing the set $\Sigma([\mathbf{A}], [\mathbf{b}])$* .

Hull of the solution set — The interval vector $[\mathbf{x}]^H = ([x]_1^H, [x]_2^H, \dots, [x]_n^H)^T$ of minimal width containing $\Sigma([\mathbf{A}], [\mathbf{b}])$ is called *the interval hull of the set $\Sigma([\mathbf{A}], [\mathbf{b}])$* or the optimal interval solution:

$$[x]_i^H = [\underline{x}_i^H, \bar{x}_i^H] \quad , \text{ where: } \begin{aligned} \underline{x}_i^H &= \min \{x_i \mid x_i \in \Sigma([\mathbf{A}], [\mathbf{b}])\} \\ \bar{x}_i^H &= \max \{x_i \mid x_i \in \Sigma([\mathbf{A}], [\mathbf{b}])\} \\ i &= 1, 2, \dots, n. \end{aligned}$$

If the interval system $[\mathbf{A}][\mathbf{x}] = [\mathbf{b}]$ can be reduced to

$$[\mathbf{x}] = [\mathbf{M}][\mathbf{x}] + [\mathbf{r}] \quad , \quad (5)$$

so that mapping $[\mathbf{M}][\mathbf{x}] + [\mathbf{r}] : I(\mathbb{R})^n \rightarrow I(\mathbb{R})^n$ is contractive with respect to usual Hausdorff distance [23], then there exists a unique fixed $[\mathbf{x}]^*$ interval of this mapping, which, by definition, fulfills

$$[\mathbf{x}]^* = [\mathbf{M}][\mathbf{x}]^* + [\mathbf{r}] \quad (6)$$

and which can be found by some iterative method [20, 33, 37, 23].

Algebraic Solution — An interval vector is called an algebraic solution of the interval equation (5) if after substituting this vector into the given equation and executing all interval operations according to the rules on interval arithmetic, an equality is obtained.

If (5) is equivalent to (3) in the sense of $\Sigma([\mathbf{A}], [\mathbf{b}])$, then

$$\Sigma([\mathbf{M}], [\mathbf{r}]) = \left\{ \tilde{x} : \tilde{x} = \widetilde{M} \cdot \tilde{x} + \tilde{r} \text{ for some } \widetilde{M} \in [\mathbf{M}], \tilde{r} \in [\mathbf{r}] \right\} \quad (7)$$

an algebraic solution $[\mathbf{x}]^*$ of (5), is also, consequently, the $\Sigma([\mathbf{A}], [\mathbf{b}])$ of the (3) as well as the hull interval $[\mathbf{x}]^H$ of $\Sigma([\mathbf{A}], [\mathbf{b}]) : [\mathbf{x}]^H \subseteq [\mathbf{x}]^*$.

Two important sources of algorithms for solving interval linear equations are [31] and [15]. The *MATLAB Intlab* toolbox have been considered for implementing algorithms such as: Interval Gaussian Elimination, Krawczyk's Method and Hansen-Blik-Rohn-Ning-Kearfott-Neumaier Method [15]. The function *verifylss* have been widely adopted for solving interval linear systems or for giving verified bounds of point systems [35, 36, 17].

2.3. Interval Probability Terminology

This section provides a brief introduction to the measure theoretic foundation of probability theory [3, 6, 21] and generalize to interval probability [1].

The correct way to define what means the term "random" is introducing the precise mathematical structure of a probability space as follows. Starting with a nonempty set denoted by Ω which have certain subsets that are interpreted as being "events".

A σ -algebra or σ -field over a set Ω is a collection \mathcal{A} of subsets of Ω that is closed under complementation and countable unions of its members with:

- $\emptyset, \Omega \in \mathcal{A}$.
- If $A \in \mathcal{A}$, then $A^c \in \mathcal{A}$.
- If $A_1, A_2, \dots \in \mathcal{A}$, then

$$\bigcup_{k=1}^{\infty} A_k, \bigcap_{k=1}^{\infty} A_k \in \mathcal{A}.$$

Here $A^c \equiv \Omega - A$ is the complement of A , where $A \in \mathcal{A}$ is called an event; and points $w \in \Omega$ are sample points.

Let \mathcal{A} be σ -algebra of subsets of Ω . $P : \mathcal{A} \mapsto [0, 1]$ is a probability measure provided:

- $P(\emptyset) = 0, P(\Omega) = 1$.
- If $A_1, A_2, \dots \in \mathcal{A}$, then

$$P\left(\bigcup_{k=1}^{\infty} A_k\right) \leq \sum_{k=1}^{\infty} P(A_k).$$

— If A_1, A_2, \dots are disjoint sets $\in \mathcal{A}$, then

$$P\left(\bigcup_{k=1}^{\infty} A_k\right) = \sum_{k=1}^{\infty} P(A_k).$$

It follows that if $A, B \in \mathcal{A}$, the

$$A \subseteq B \Rightarrow P(A) \leq P(B),$$

where $P(A)$ is a probability of the event A .

Probability Space — A triple (Ω, \mathcal{A}, P) is called probability space provided Ω is any set, \mathcal{A} is a σ -algebra of subsets of Ω , and P is a probability measure on \mathcal{A} . A property which is true except for an event of probability zero is said to hold almost surely.

Considering the case of a finitely generated algebra \mathcal{A} based on a sample space Ω [1], Ω is finite, and \mathcal{A} is the power set of $\Omega = \{\omega_1, \omega_2, \dots, \omega_n\}$. The terminology for every probability measure in the usual sense, i.e. every set function $p(\cdot)$ satisfying Kolmogorov's Axioms is called a *classical probability*. The set of all classical probabilities on the measurable space (Ω, \mathcal{A}) is denoted by $\mathcal{K}(\Omega, \mathcal{A})$. Axioms for interval-valued probabilities $p(\cdot) = [l(\cdot), u(\cdot)]$ can be obtained by looking at the relation between the non-additive set-function $l(\cdot)$ and $u(\cdot)$ and in accordance with the set of classical probabilities, where:

$$\begin{aligned} p(\cdot) : \mathcal{A} &\longrightarrow \mathcal{X}_0 := \{[l, u] | 0 \leq l \leq u \leq 1\} \\ A \in \mathcal{A} &\longmapsto p(A) = [l(A), u(A)] \end{aligned}$$

with

$$\mathcal{M} = \{p(\cdot) \in \mathcal{K}(\Omega, \mathcal{A}) | l(A) \leq p(A) \leq u(A), \forall A \in \mathcal{A}\} \neq \emptyset \quad (8)$$

and

$$\inf_{p(\cdot) \in \mathcal{M}} p(A) = l(A), \quad \sup_{p(\cdot) \in \mathcal{M}} p(A) = u(A). \quad (9)$$

Such $p(\cdot)$, and the corresponding set functions $l(\cdot)$ and $u(\cdot)$, are called lower and upper probability [16], envelopes [39, 9], coherent probability [40] and F-probability [42, 43, 41]. Weichselberger's terminology calls \mathcal{M} structure. Note that, by (9), there is a one-to-one correspondence between $p(\cdot)$ and the structure \mathcal{M} .

More general sets of classical probabilities are obtained by the theory of coherent previsions [40]. By the lower envelope theorem ([40], p.134)

and the fact that classical expectation and classical probabilities uniquely correspond with each other, the definition of coherence can be rewritten in a way similar to (9). Since Walley [40] did not coin a name for the resulting set of classical probabilities, it will be called structure, too.

3. ICTMC: Interval Continuous-Time Markov Chain

A model space representing uncertainties or imprecision can explicitly be expressed by an interval model, i.e. model in which there are parameters represented by intervals instead of real numbers. Therefore, an interval model is, actually, a family, a class or a space of models; and a model can be viewed as a particular interval model in which interval widths are zero. The Markov (memoryless) property to a ICTMC (Interval Continuous-Time Markov Chain) is given by:

$$\begin{aligned} [\mathbf{P}] \{ [\mathbf{X}](\tilde{t}_{k+1}) = x_{k+1} | [\mathbf{X}](\tilde{t}_k) = x_k, [\mathbf{X}](\tilde{t}_{k-1}) = x_{k-1}, \dots, [\mathbf{X}](\tilde{t}_0) = x_0 \} \\ = \mathbf{P} \{ [\mathbf{X}](\tilde{t}_{k+1}) = x_{k+1} | [\mathbf{X}](\tilde{t}_k) = x_k \}, \end{aligned}$$

where $\tilde{t}_0 \leq \tilde{t}_1 \leq \dots \leq \tilde{t}_k \leq \tilde{t}_{k+1}$, $\forall \tilde{t}_i \in [t_i]$, $[t_i] = [\tilde{t}_i - \Delta t_i, \tilde{t}_i + \Delta t_i] \equiv \tilde{t}_i \pm \Delta t_i$, $\tilde{t}_i = \text{mid}[i]$, $\Delta t_i = \text{rad}[t_i]$, $i = 0, 1, 2, \dots, k + 1$.

The $[\mathbf{X}](\tilde{t}_{k+1})$ value depends only on x_k and not on any past state history (no state memory). The time spent in the current state is irrelevant to determining the next state (no age memory) [5].

The study of interval continuous-time Markov chain considers interval rates at which events (state transitions) occur. This allows specifying an interval model and carrying out the interval arithmetic analysis.

Let χ denote the set of states of continuous time Markovian stochastic process, the transition probability between states $i, j \in \chi$ is specified by an interval matrix $[\mathbf{P}](\chi)$, whose entry, $[p]_{ij}(\tilde{t})$, $\forall \tilde{t} \in [t]$, is the interval probability of a transition from i to j within a time interval of duration \tilde{t} for all possible $\tilde{t} \in [t]$. Hence, every value within the interval $[p]_{ij}(\tilde{t})$ is possible probability transition. One possible solution is introducing information regarding the interval rate at which various state transitions (events) may occur at any time.

The time-dependent transition interval probabilities are defined as follows:

$$\begin{aligned} [p]_{ij}([s], [t]) \equiv P[X]([t]) = j | [X]([s]) = i, \quad \forall \tilde{s} \in [s], \forall \tilde{t} \in [t], \tilde{s} \leq \tilde{t}, \quad (10) \\ [s] = [\tilde{s} - \Delta s, \tilde{s} + \Delta s] \equiv \tilde{s} \pm \Delta s, \end{aligned}$$

$$[t] = [\bar{t} - \Delta t, \bar{t} + \Delta t] \equiv \bar{t} \pm \Delta t.$$

The transition interval functions $[p]_{ij}([s], [t])$ is a function of the time instants $\bar{s} \in [s]$ and $\bar{t} \in [t]$ satisfying the interval continuous-time Chapman-Kolmogorov equation

$$[p]_{ij}([s], [t]) = \sum_{all\ r} [p]_{ir}([s], [u])[p]_{rj}([u], [t]),$$

$$\forall \bar{s} \in [s], \forall \bar{u} \in [u], \forall \bar{t} \in [t], \bar{s} \leq \bar{u} \leq \bar{t}. \quad (11)$$

To rewrite (11) in interval matrix form, we define:

$$[\mathbf{H}]([s], [t]) \equiv [p]_{ij}([s], [t]), \quad i, j = 0, 1, 2, \dots \quad (12)$$

and observe that $[\mathbf{H}]([s], [s]) = \mathbf{I}$, $\forall \bar{s} \in [s]$ (the identity matrix). Then from (11):

$$[\mathbf{H}]([s], [t]) = [\mathbf{H}]([s], [u])[\mathbf{H}]([u], [t]),$$

$$\forall \bar{s} \in [s], \forall \bar{u} \in [u], \forall \bar{t} \in [t] \quad \bar{s} \leq \bar{u} \leq \bar{t}. \quad (13)$$

The interval Chapman-Kolmogorov equation is the extended general relationship for Markov memoryless property.

Homogeneous continuous-time Markov chain is defined when all transition function $[p]_{ij}([s], [t])$, depicted in (10), are independent of the absolute time instants $[s]$, $[t]$ and depend only on the difference $[\tau] = [s] - [t]$. For homogeneous continuous-time systems, the Kolmogorov's forward differential equation (14) is obtained

$$\frac{d[\mathbf{P}]([\tau])}{dt} = \mathbf{P}([\tau])[\mathbf{Q}]. \quad (14)$$

With the following initial conditions (assuming that a state transition from any i to $j \neq i$ cannot occur in zero time):

$$[p]_{ij}(0) = \begin{cases} 1 & \text{if } j = i, \\ 0 & \text{if } j \neq i. \end{cases} \quad (15)$$

The solution of (14) is of the form

$$[\mathbf{P}]([\tau]) = e^{[\mathbf{Q}][\tau]}. \quad (16)$$

Defining interval state probabilities

$$[\pi]_j(t) \equiv [\mathbf{P}][\mathbf{X}([t]) = j] \quad (17)$$

yields an *interval state probability vector*

$$[\pi](t) = [[\pi]_0(t), [\pi]_1(t), [\pi]_2(t), \dots].$$

This is a new interval vector whose dimension is given by the dimension of the state space of the chain (not necessarily finite).

For many practical applications, the system's steady state behavior is of interest. In other words, the system is observed after "long-run", that is,

$$[\pi]_j = \lim_{t \rightarrow \infty} [\pi]_j(t). \quad (18)$$

If π_j exists for some state j , it is called a *stead-state, equilibrium, or stationary state interval probability vector*

$$[\pi] = [[\pi]_0, [\pi]_1, [\pi]_2, \dots].$$

If the limit (18) exists, it follows that

$$\lim_{t \rightarrow \infty} \frac{d[\pi](t)}{dt} \rightarrow 0$$

does not depend on t . Then, the interval steady state differential equation reduces to the algebraic equation

$$[\pi][\mathbf{Q}] = \mathbf{0}. \quad (19)$$

The limit (18) is independent on the initial state interval probability vector. Moreover, the interval vector $[\pi][\mathbf{Q}] = \mathbf{0}$ is determined by solving an interval linear system of equations, where the left-hand side $[\mathbf{Q}] \in I(\mathbb{R})^{n \times n}$ is the coefficient matrix and right-hand side of the equation is $\mathbf{0} = (0, 0, 0, \dots, 0)^t \in \mathbb{R}^n$. Hence, $[\pi][\mathbf{Q}] = \mathbf{0}$ is defined as a family of linear equations such that

$$\tilde{\pi} \cdot \tilde{\mathbf{Q}} = 0 \quad (\tilde{\mathbf{Q}} \in [\mathbf{Q}]) \quad \text{under the condition} \quad \tilde{\pi} \cdot \mathbf{1} = 1,$$

or rewriting

$$[\pi][\mathbf{Q}] = \mathbf{0} \quad (20)$$

under the condition $\sum_{\text{all } j} [\pi]_j = 1$.

Thus, we consider a linear system of equations in which the coefficients are intervals. We are interested in enclosures for the solution set of (20), given by

$$\Sigma([\mathbf{A}], b) = \{\bar{\pi} \in \mathbb{R}^n \mid \bar{\pi} \cdot \tilde{\mathbf{A}} = b\}$$

for some $\tilde{\mathbf{A}} \in [\mathbf{A}]$ and $b = (0, 0, 0, \dots, 1)^t$.

where, as usual, $[\mathbf{A}]$ is obtained by eliminating one equation of (20) and keeping the normalization condition $\bar{\pi} \cdot \mathbf{1} = 1$.

Both direct and iterative methods are considered for finding an interval vector containing $\Sigma([\mathbf{A}], b)$. The enclosure quality (overestimation) of $\Sigma([\mathbf{A}], b)$ depends on the underlying system.

4. IGSPN: Interval GSPN

IGSPN extends the GSPN model in order to introduce the interval analysis. The GSPN is a particular timed PN that incorporate both stochastic timed transitions (drawn as white boxes) and immediate transitions (drawn as thin black bars). Timed transitions have an exponentially distributed firing time and immediate transitions fire in zero time. GSPNs were originally defined in [28] and was later improved [26, 27, 7, 4].

An IGSPN is also defined as a nine-tuple as described bellow:

$$IGSPN = (P, T, I, O, H, G, [W], \Pi, M_0)$$

— $P = \{p_1, \dots, p_j, \dots, p_m\}$ is the set of places, where *p_j denotes the set of input transitions of place p_j and p_j^* denotes the set of output transitions of place p_j ,

— $T = \{t_1, \dots, t_i, \dots, t_n\}$ is the set of transitions, where *t_i denotes the set of input places of transition t_i , t_i^* denotes the set of output places of transition t_i , ${}^{\circ}t_i$ denotes the set of inhibitor places of transition t_i . $T = T_{im} \cup T_t$, $T_{im} \cap T_t = \emptyset$, T_{im} is the set of immediate transitions and T_t is the set of timed transitions,

— $I : P \times T \rightarrow \mathbb{N}$ is mapping of ordered pairs to natural numbers, where $(p_j, t_i) \in P \times T \Leftrightarrow p_j \in {}^*t_i$. $I(p_j, t_i)$ represents the arc weight that connects p_j to t_i ,

— $O : T \times P \rightarrow \mathbb{N}$ is mapping of ordered pairs to natural numbers, where $(t_i, p_j) \in T \times P \Leftrightarrow p_j \in t_i^*$. $O(t_i, p_j)$ represents the arc weight that connects t_i to p_j .

— $H : P \times T \rightarrow \mathbb{N}$ is mapping of ordered pairs to natural numbers, where $(p_j, t_i) \in P \times T \Leftarrow p_j \in {}^o t_i$. $H(p_j, t_i)$ represents the inhibitor arc weight that connects p_j to t_i ,

— $G \in (T \rightarrow (\mathbb{N}^p \rightarrow \{true, false\}))$ is partial function that assigns to transitions a place marking expression (binary relations $=, \neq, <, \leq, >, \geq$) that could be evaluated to *true* or *false*,

— $[W] : T \rightarrow I(\mathbb{R}) = \begin{cases} [r]_i \text{ is the rate interval of the negative} \\ \text{exponential pdf if } t_i \in T_t, \\ [w]_i \text{ is the interval weight if } t_i \in T_{im}. \end{cases}$

— $\Pi : T \rightarrow \mathbb{N} = \begin{cases} 0 & \text{if } t_i \in T_t \\ \geq 1 & \text{if } t_i \in T_{im} \end{cases}$ is a mapping of transitions to natural numbers, that denotes the transition firing priority, in which higher the number higher the respective firing priority,

— $M_0 : P \rightarrow \mathbb{N}$ is a mapping of places to natural number that denotes the net initial marking.

If only point intervals are defined for exponential and immediate transitions, we have a GSPN model.

4.1. IGSPN Analysis

An IGSPN is a GSPN extension, where interval parameters should be considered in order to obtain the interval infinitesimal generator of ICTMC. The classical algorithms found in literature [2] are naturally adapted to take into account interval coefficients of IGSPN model. The analysis of dependability using IGSPN is accomplished in four subtasks:

- generation of the IERG (Interval Extended Reachability Graph),
- elimination of vanishing markings and the corresponding state transitions,
- interval steady-state analysis,
- computation of measures. Standard measures such as the average number of tokens in places and the throughput of timed transitions are computed by interval arithmetic.

From a given IGSPN, an interval extended reachability graph (IERG) is generated containing markings as nodes and interval stochastic information attached to arcs so as to relate markings to each other. IGSPN reachability graph is a directed graph $RG(IGSPN) = (V, E)$, where $V = RS(IGSPN)$ and $E = \{(\mathbf{m}, t, \mathbf{m}') \mid \mathbf{m}, \mathbf{m}' \in RS(IGSPN) \text{ and } \mathbf{m} \xrightarrow{t} \mathbf{m}'\}$ are the set of nodes and edges, respectively. If an IGSPN model is bounded, the $RG(IGSPN)$ is finite and it can be constructed, for example, based on

Algorithm 5.1 (Computation of the Reachability Graph) p. 61 from [13].

The elimination of vanishing markings is a step for generating the ICTMC from a given IGSPN. Once the IERG has been generated, it is transformed into a ICTMC by the use of matrix algorithms [2].

The markings $\mathcal{M}k = \mathcal{V} \cup \mathcal{T}$ in reachability set of a IGSPN are partitioned into two sets, the vanishing markings \mathcal{V} and the tangible markings \mathcal{T} . Let:

$$[\mathbf{P}]^{\mathcal{V}} = [\mathbf{P}]^{\mathcal{V}\mathcal{V}} \mid [\mathbf{P}]^{\mathcal{V}\mathcal{T}} \quad (21)$$

denote an interval matrix, where:

- $[\mathbf{P}]^{\mathcal{V}\mathcal{V}}$ - interval transition probabilities between vanishing markings,
- $[\mathbf{P}]^{\mathcal{V}\mathcal{T}}$ - interval transition probabilities from vanishing markings to the tangible markings.

Furthermore, let:

$$[\mathbf{U}]^{\mathcal{T}} = [\mathbf{U}]^{\mathcal{T}\mathcal{V}} \mid [\mathbf{U}]^{\mathcal{T}\mathcal{T}} \quad (22)$$

denote an interval matrix, where:

- $[\mathbf{U}]^{\mathcal{T}\mathcal{V}}$ - interval transition rates from tangible to vanishing markings;
- $[\mathbf{U}]^{\mathcal{T}\mathcal{T}}$ - interval transition rates between tangible markings.

The same information as contained in the IERG is provided by $[\mathbf{P}]^{\mathcal{V}}$ together with $[\mathbf{U}]^{\mathcal{T}}$.

Now, the interval rate matrix $[\mathbf{U}]$ is obtained. This matrix has dimension $|\mathcal{T}| \times |\mathcal{T}|$, where \mathcal{T} denotes the set of tangible markings.

$$[\mathbf{U}] = [\mathbf{U}]^{\mathcal{T}\mathcal{T}} + [\mathbf{U}]^{\mathcal{T}\mathcal{V}}(1 - [\mathbf{P}]^{\mathcal{V}\mathcal{V}})^{-1}[\mathbf{P}]^{\mathcal{V}\mathcal{T}}. \quad (23)$$

The interval infinitesimal generator matrix is $[\mathbf{Q}] = [q]_{ij}$, where its entries are given by:

$$[q]_{ij} = \begin{cases} [u]_{ij} & \text{if } i \neq j, \\ -\sum_{\substack{k \in \mathcal{T}, \\ k \neq i}} [u]_{ik} & \text{if } i = j. \end{cases} \quad (24)$$

where \mathcal{T} denotes the set of tangible markings.

The state-stationary solution of the ICTMC (Interval Continuous Time Markov Chain) model underlying IGSPN is obtained by solving the interval linear equations system with as many equations as the number of tangible markings

$$\begin{cases} [\pi] \cdot [\mathbf{Q}] = \mathbf{0}, \\ \sum_{M \in \mathcal{T}} [\pi](M) = 1. \end{cases} \quad (25)$$

$[\pi]$ is the interval vector for the equilibrium *pmf* (probability mass function) over the reachable tangible markings, and $[\pi](M)$ represents the interval steady-state probability of a given tangible marking M .

Once the interval generator matrix $[Q]$ of the ICTMC associated with a IGSPN model has been derived, we can start with the IGSPN steady state analysis to yield interval dependability measures.

5. Case Study

In this section, the availability analysis of two network devices widely used in computer networks, namely a Multiplexer ADM and a Multiplexer SDH, are carried out. These devices are made up of several components (memory, source power, switch, mux controller, tributary) that have either no redundancy or hot standby redundancy (see Section 2.1). Additionally, all these components (with redundancy or not) are composed in a serial manner, which means that if just one of them fails, then the whole composition fails.

The IGSPN models of these devices are shown in Figures 2 and 3. Each component has two parameters, namely *MTTF_X* (Mean Time To Fail) and *MTTR_X* (Mean Time To Recover) that represent delays associated to transitions *Failure_X* and *Repair_X*, respectively. The label “*X*” must be instantiated with the component name, e.g., “*MADM_Source_ON*” and “*MADM_Source_OFF*”. The model places *X_ON* and *X_OFF* are the activity and inactivity of a given component, respectively.

The aggregate models (bottom right of Figure 2 and bottom left of Figure 3) have only immediate transitions. They provide support for evaluating both the reliability and availability of systems, since they explicitly represent system’s state. In these aggregate models, one token in places (*MADM_Up*, *MDH_PSW_Up*) and (*MADM_Down*, *MSDH_PSW_Down*) depicts the operational and failure states, respectively. All transitions have enabling functions (guarded functions) attached to them. These functions are considered for combining components taking a specific into account a specific structure. Tables 1 and 2 presents the guard expressions associated to the enabling functions of the aggregate model.

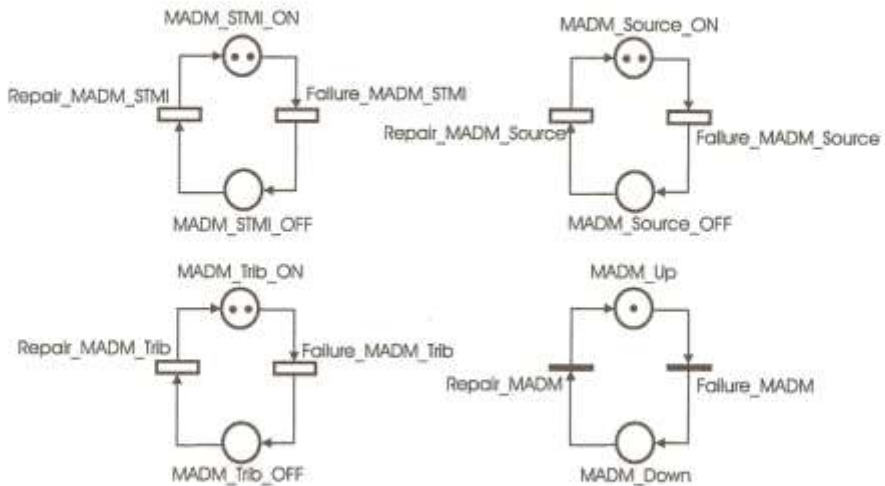


Figure 2: The IGSPN MUX ADM model

Transition	Guard expression
Repair_MADM	$(\#MADM_STMI_ON > 0)$ and $(\#MADM_Source_ON > 0)$ and $\#MSDH_Trib_ON > 0)$
Failure_MADM	$(\#MADM_STMI_ON = 0)$ or $(\#MADM_Source_ON = 0)$ or $(\#PMSDH_Trib_ON = 0)$

Table 1: Guard expressions for the aggregated MUX ADM model

Tables 3 and 4 depict exponential times assigned to transitions of the IGSPN model. It is worth observing that common time intervals ($[7.4, 8.6]$ and $[7.8, 8.2]$ in hours, MUX ADM and MUX SDH, respectively) have been associated to transitions of kind “Repair_X” of each model. These time intervals are related to the maintenance policy adopted in order to provide a contract service quality agreement.

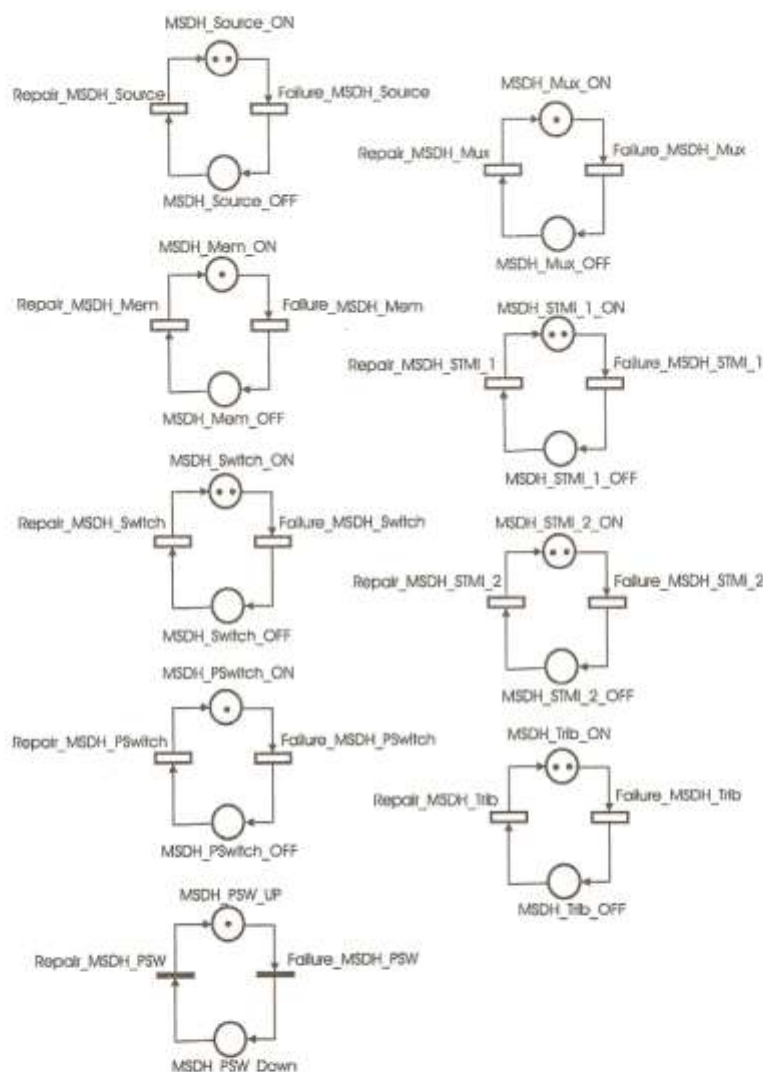


Figure 3: The IGSPN model of the multiplexer SDH

The dependability analysis is carried out with the IGSPN prototype tool. The IGSPN analysis with point intervals corresponds to the classical GSPN models analysis, nevertheless, the obtained results are verified; which means that rounding and truncation process have been consistently considered by interval arithmetic theory framework. The IGSPN analysis (with thick in-

Transition	Guard expression
Repair_MSDH_PSW	$(\neq \text{MSDH_Source_ON} > 0)$ $\text{and } (\neq \text{MSDH_Mux_ON} > 0)$ $\text{and } (\neq \text{MSDH_Mem_ON} > 0)$ $\text{and } (\neq \text{MSDH_STMI.1_ON} > 0)$ $\text{and } (\neq \text{MSDH_Switch_ON} > 0)$ $\text{and } (\neq \text{MSDH_STMI.2_ON} > 0)$ $\text{and } (\neq \text{MSDH_PSwitch_ON} > 0)$ $\text{and } (\neq \text{MSDH_Trib_ON} > 0)$
Failure_MSDH_PSW	$(\neq \text{MSDH_Source_ON} = 0)$ $\text{or } (\neq \text{MSDH_Mux_ON} = 0)$ $\text{or } (\neq \text{MSDH_Mem_ON} = 0)$ $\text{or } (\neq \text{MSDH_STMI.1_ON} = 0)$ $\text{or } (\neq \text{MSDH_Switch_ON} = 0)$ $\text{or } (\neq \text{MSDH_STMI.2_ON} = 0)$ $\text{or } (\neq \text{MSDH_PSwitch_ON} = 0)$ $\text{or } (\neq \text{MSDH_Trib_ON} = 0)$

Table 2: Guard expressions for the aggregated MUX SDH GPT with Pswitch model

Transition	Time (h)	Interpretation
Repair_MADM_STMI	[7.4, 8.6]	Repair STMI
Failure_MADM_STMI	248000	Failure STMI
Repair_MADM_Source	[7.4, 8.6]	Repair Source
Failure_MADM_Source	657000	Failure Source
Repair_MADM_Trib	[7.4, 8.6]	Repair Trib
Failure_MADM_Trib	271000	Failure Trib

Table 3: Exponential transition times for the MUX ADM

tervals) depicts the effects of variabilities in exponential transition rates and immediate transition parameters to dependability indices.

Tables 5 and 6 present the availability evaluation results related to the MUX ADM and MUX SDH GPT with Pswitch, respectively, considering the proposed SPN (IGSPN). If every delay related to timed transition and weights associated to immediate transition are all thin intervals, that is, they are equivalent to point values, the proved results are comparable to those obtained by GSPN models. However, it should be stressed that those

Transition	Time (h)	Interpretation
Repair_MSDH_Source	[7.8, 8.2]	Repair Source
Failure_MSDH_Source	657000	Failure Source
Repair_MSDH_Mux	[7.8, 8.2]	Repair Mux controller
Failure_MSDH_Mux	350000	Failure Mux controller
Repair_MSDH_Mem	[7.8, 8.2]	Repair Memory
Failure_MSDH_Mem	438000	Failure Memory
Repair_MSDH_STMI_1	[7.8, 8.2]	Repair STMI ₁
Failure_MSDH_STMI_1	762000	Failure STMI ₁
Repair_MSDH_Switch	[7.8, 8.2]	Repair Switch
Failure_MSDH_Switch	657000	Failure Switch
Repair_MSDH_STMI_2	[7.8, 8.2]	Repair STMI ₂
Failure_MSDH_STMI_2	762000	Failure STMI ₂
Repair_MSDH_PSwitch	[7.8, 8.2]	Repair PSwitch
Failure_MSDH_PSwitch	1490000	Failure PSwitch
Repair_MSDH_Trib	[7.8, 8.2]	Repair Tributary
Failure_MSDH_Trib	438000	Failure Tributary

Table 4: Exponential transition times for the MUX SDH GPT with Pswitch

are verified results instead, since rounding and truncation are dealt with interval arithmetic. Therefore, interval metric results provided guaranty the inclusion of actual point values. The system availability are:

$$\langle 0.99999999587965, 0.00000000000001 \rangle$$

and

$$\langle .99995446712728, 0.00000000000022 \rangle,$$

that is, the actual point value are within $[0.99999999587964, 0.99999999587965]$ and $[0.99995446712706, 0.99995446712750]$, respectively, that represent both upper and lower verified bounds.

When considering that the mean time to repair (MTTR) are not precisely known, but known to belong to the intervals $[7.4, 8.6]$ and $[7.8, 8.2]$ in hours, the MUX ADM and MUX SDH the availability are given by

$$\langle 0.99984112578944, 0.00015887421057 \rangle$$

and

$$\langle 0.99993565053080, 0.00006434946921 \rangle,$$

Exponential time interval	Display of intervals/Value
Thin	mid/rad (0.99999999587965, 0.00000000000001) inf/sup [0.99999999587964, 0.99999999587965]
Thick	mid/rad (0.99984112578944, 0.00015887421057) inf/sup [0.99968225157888, 1.00000000000000]

Table 5: System availability for MUX ADM model (thin intervals considered are midpoint of thick intervals)

Exponential time interval (<i>type</i>)	Display of interval / (<i>System availability value</i>)
Thin	mid/rad (0.99995446712728, 0.000000000000022) inf/sup [0.99995446712706, 0.99995446712750]
Thick	mid/rad (0.99993565053080, 0.00006434946921) inf/sup [0.99987130106160, 1.00000000000000]

Table 6: System availability for MUX SDH GPT with Pswitch (thin intervals considered are midpoint of thick intervals)

respectively. It means that all possible availability results are guaranteed to be within intervals [0.99968225157888, 1.00000000000000] and [0.99987130106160, 1.00000000000000], for MUX ADM and MUX SDH, respectively.

The obtained results highlight the variability effects (7.5% MUX ADM and 2.5% MUX SDH) related to the components MTTRs on the system's availability. In such a case, the reader may observe the expected reduction on the system's availability.

Once again, it is important to stress the appropriate understanding of the real intervals vector result obtained from a "system of interval linear equations". The number of markings are 46 (27T+19V, T=Tangible, V=Vanishing) for MUX ADM model and 2152 (1944T+208V) for MUX SDH GPT with Pswitch model. Thus, we consider a interval linear system of equations with the number of tangible markings for each model. The real intervals' vector provided asserts that the intervals contain the elements of the exact solution [15]. The Krawczyk's method have been considered, through the function *verifyflls* of the *Intlab* toolbox, for solving the interval linear systems (25). This function provides outer bounds solutions.

6. Conclusions

In this paper, IGSPN has been adopted as a method for modeling and dependability analysis in which the exponential rates are known to belong to some given positive real intervals. The IGSPN is mainly applied to model situations in which input data is known with certain accuracy level. Rates as well as timing uncertainties might be subjectively specified as intervals. This framework provides a way to formalize and study problems related to the presence of such uncertainties that may include data errors occurred during data measurements and rounding errors generated during calculations. The proposed model and the related analysis method allow dependability analysis considering simultaneous variability of parameters. As immediate consequence, the IGSPN analysis may be useful for engineers and technicians as a tool for decision-making. As future works, methods for interval transient analysis and simulation should be considered. Furthermore, other case studies should also be taken into account.

References

- [1] T. Augustin, On the suboptimality of the generalized bayes rule and robust bayesian procedures from the decision theoretic point of view: A cautionary note on updating imprecise priors, *ISIPTA '03* (2003), 31-45.
- [2] G. Bolch, S. Greiner, K.S. Trivedi, *Queueing Networks and Markov Chains, Modeling and Performance Evaluation with Computer Science Applications*, Wiley Interscience Publication (2006).
- [3] P. Bremaud, *An Introduction to Probabilistic Modeling*, Springer Verlag (1988).
- [4] Ed Brinksma, Editor, Lectures on formal methods and performance analysis, Revised Lectures – First EEF Summer School on Trends in Computer Science. Berg en Dal, The Netherlands, July 3-7, 2000. 9ed. Springer (2001),
- [5] C.G. Cassandras, S. Lafortune, *Introduction to Discret Event Systems*, Kluwer Academic Publishers, Boston-Dordrecht-London (1999).
- [6] K.L. Chung, *Elementary Probability Theory with Stochastic Processes*, Springer (1975).

- [7] G. Ciardo, *Analysis of Large Stochastic Petri Net Models*, Ph.D. Thesis, Duke University (1989).
- [8] D. Crowe, A. Feinberg, *Design for Reliability*, CRC Press (2001).
- [9] D. Denneberg, *Non-Additive Measure and Integral*, Kluwer Academic Publishers, Dordrecht (1994).
- [10] J.B. Dugan, K.S. Trivedi, R.M. Geist, V.F. Nicola, Extended stochastic Petri nets - applications and analysis, In: *Performance'84* (Ed-s: R. Dumke), Elsevier (1984), 507-519.
- [11] J.C. Geffroy, G. Motet, *Design of Dependable Computing Systems*, Kluwer Academic Publishers, The Netherlands (2002).
- [12] R. German, *Performance Analysis of Communicating Systems - Modeling with Non-Markovian Stochastic Petri Nets*, John Wiley and Sons (2000).
- [13] C. Girault, R. Valk, *Petri Nets for Systems Engineering. A Guide to Modeling, Verification, and Applications*, Springer (2003).
- [14] E.R. Hansen, G.W. Walster, *Global Optimization Using Interval Analysis*, Second Edition, Revised and Expanded, Marcel Dekker, Inc., New York (2004).
- [15] G.I. Hargreaves, Interval analysis in *MATLAB*, *Numerical Analysis Report*, No. 416, University of Manchester. Reports available from: Department of Mathematics <http://www.ma.man.ac.uk/mccm>, <http://www.ma.man.ac.uk/nareports>, Manchester M13 9PL, England, December (2002).
- [16] P.J. Huber, V. Strassen, Minimax tests and the Neyman-Pearson lemma for capacities, *Annals of Statistics*, **1** (1973), 251-263.
- [17] *Intlab 5.3*, <http://www.Ti3.Tu-harburg.De/rump/intlab/> (2007).
- [18] V. Kreinovich G. Heindl, A. Lakeyev, Solving linear interval is np-hard even if we exclude overflow and underflow, *Reliable Computing*, **4** (1998), 383-388.
- [19] U.W. Kulish, W.L. Miranker, The arithmetic of the digital computers, *A New Approach. SIAM Review*, **28**, No. 1 (1986).

- [20] L. Kupriyanova, Inner estimation of the united solution set of interval linear algebraic system, *Reliable Computing*, **1** (1995), 15-31.
- [21] J. Lamperti, *Probability: A Survey of the Mathematical Theory*, W.A. Benjamin Inc., New York (1966).
- [22] J.C. Laprie, Dependability, basic concepts and associated terminology, In: *Dependable Computing and Fault-Tolerant Systems*, **5**, Springer-Verlag (1992).
- [23] M. Lyashko, The optimal solution of an interval system of linear algebraic equations, *Reliable Computing*, **11** (2005), 105-127.
- [24] S. Majumdar, J. Lüthi, G. Haring, R. Ramadoss, Characterization and analysis of software and computer systems with uncertainties and variabilities, In: *Performance Engineering. LNCS 2047* (Ed. R. Dumke), Springer-Verlag, Berlin, Heidelberg (2001), 202-221.
- [25] M. Malhotra, J. Muppala, K.S. Trivedi, Stiffness-tolerant methods for transient analysis of stiff Markov chains, *International Journal on Microelectronics and Reliability*, **34**, No. 11 (1994), 1825-1841.
- [26] M.A. Marsan, G. Balbo, G. Chiola, G. Conte, Generalized stochastic Petri nets revisited: random switches and priorities, In: *Proceedings International Workshop on Petri Nets and Performance Models - PNPM87*, IEEE Computer Society (1987), 44-53.
- [27] M.A. Marsan, G. Balbo, G. Chiola, G. Conte, A. Cumani, Generalized stochastic Petri nets: A definition at the net level and its implications, *IEEE Transactions on Software Engineering*, **19** (1993), 89-107.
- [28] M.A. Marsan, G. Balbo, G. Conte, A class of generalized stochastic Petri nets, *ACM Transactions Computer Systems*, **2** (1984), 93-122
- [29] M. Ajmone Marsan, G. Balbo, G. Conte, S. Donatelli, G. Franceschinis, *Modelling with Generalized Stochastic Petri Nets*, Wiley Series in Parallel Computing, John Wiley and Sons (1995).
- [30] R.E. Moore, *Interval Analysis*, Prentice Hall, Englewood Cliffs, NJ, USA (1966).
- [31] A. Neumaier, *Interval Methods for Systems of Equations*, Cambridge University Press, Cambridge (1990).

- [32] D.K. Pradhan, *Fault Tolerant Computer System Design*, Prentice Hall Advanced Computing and Telecommunications Series, Upper Saddle River, NJ (1996).
- [33] H. Ratschek, W. Sauer, Linear interval equations, *Computing*, **28** (1982), 105-115.
- [34] S.M. Rump, *Intlab - interval laboratory*, Technical Report, Universität Haburg-Hamburg, Hamburg, Germany (1998).
- [35] S.M. Rump, Fast and parallel interval arithmetic, *BIT*, **39**, No. 3 (1999), 534-554.
- [36] S.M. Rump, Intval labatory, In: *Developments in Reliable Computing* (Ed. Tibor Csends), Kluwer, Dordrecht, Netherlands (1999), 77-105.
- [37] S.P. Shary, Algebraic approach in the "outer problem" for interval linear equations, *Reliable Computing*, **3** (1997), 103-135.
- [38] S.P. Shary, Outer estimation of generalized solution sets to interval linear systems, *Reliable Computing*, **5** (1999), 323-335.
- [39] P. Walley, T.L. Fine, Towards a frequentist theory of upper and lower probability, *Annals of Statistics*, **10**, No. 3 (1982), 741-761.
- [40] P. Walley, *Statistical Reasoning with Imprecise Probabilities*, Chapman and Hall, London-New York-Tokyo-Melbourne-Madras (1991).
- [41] K. Weichselberger, *Elementare Grundbegriffe Einer Allgemeineren Wahrscheinlichkeitsrechnung*, Volume I.
- [42] K. Weichselberger, Axiomatic foundations of the theory of interval-probability, In: *Proceedings of the Second Gauss Symposion, Section B*, Mammitzsch and Schneeweiss (Ed. B. De Gruyter) (1995), 47-64.
- [43] K. Weichselberger, The theory of interval-probability as a unifying concept for uncertainty, *International Journal of Approximate Reasoning*, **24** (2000), 149-170.
- [44] W.M. Zuberek, Timed Petri nets - definitions, properties, and applications, *Microelectronics and Reliability*, **31**, No. 4 (1991), 627-644.